Monolithic isogeometric analysis of two fluid flow: towards energy stability

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ABSTRACT

A two-fluid formulation is presented that is a continuation of the isogeometric work presented in [1]. The new formulation exhibits correct kinetic and potential energy evolution, that is guaranteed decay of energy in the absence of an external energy source. Achieving this is not trivial, in fact most practical time stepping approaches have a hidden instability stemming from the decoupling of flow and interface solves. See [2] for a clear demonstration of the issue.

Correct energy behavior is achieved by solving the interface evolution with special care. For starters the flow and interface evolution are solved monolithically [2]. Additionally, essential constraints on the interface evolution are enforced using global Lagrange multipliers.

For the monolithic coupling of flow and interface a special level–set formulation is used [3]. In this level–set formulation the difficult non–linear Eikonal problem is translated to a simple linear projection problem.

The formulation with Lagrange multipliers is solved with a Quasi-newton method. This method partially decouples the constraints from the rest of the problem. This results in a favorable matrix structure and the ability to solve the constraint to a strict tolerance without increasing the global iteration count.

Divergence conforming NURBS spatial discretization is adopted [4]. This avoids ambiguities with regard to mass and volume conservation, which are equivalent in the incompressible case.

The energy properties of the proposed method are verified with the tried and tested dambreak problem. Examining the convergence of the energy evolution demonstrates the potential of the proposed formulation. The diffusion is chosen such that convective stabilization is not required. The correct energy behavior of stabilized formulation for the single fluid case is discussed [5].

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