Low-rank solvers for Isogeometric analysis in PDE-constrained optimization

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ABSTRACT

Isogeometric analysis has become one of the most popular methods for the discretization of partial differential equations motivated by the use of NURBS for geometric representations in industry and science. A crucial challenge lies in the solution of the discretized equations, which we discuss in this talk with a particular focus on PDE-constrained optimization subject to a PDE discretized using IGA.

The discretization results in a system of large mass and stiffness matrices, which are typically very costly to assemble. To reduce the computing time and storage requirements low-rank tensor methods as proposed in [1] have become a promising tool. We present a framework for the assembly of these matrices M in low-rank form as the sum of a small number of Kronecker products $M = \sum_{i=1}^{n} \bigotimes_{d=1}^{D} M_i^{(d)}$, where D is the geometry's dimension (2 or 3) and n is determined by the chosen size of the low rank approximation. For assembly of the smaller matrices $M_i^{(d)}$ only univariate integration in the corresponding geometric direction d is required.

The resulting low rank Kronecker product structure of the mass and stiffness matrices can be used to solve a PDE-constrained optimization problem without assembly of the actual system matrices. We present a framework which preserves and exploits the attained Kronecker product format using the *amen block solve* algorithm from the tensor train toolbox [2] in MATLAB to efficiently solve the corresponding KKT system of the optimization problem. We illustrate the performance of the method on various examples.

REFERENCES

[1] A. Mantzaflaris, B. N. Khoromskij, and U. Langer, Low rank tensor methods in Galerkin-based isogeometric analysis, *Comp. Methods Appl. Mech. Engrg.*, Vol. **316**, pp. 1062-1085 (2017).

[2] I. V. Oseledets, Tensor-Train Decomposition, *SIAM J. Sci. Comput.*, Vol. **33**(5), pp. 2295-2317 (2011)