Correct Energy Evolution of Stabilized Methods: Dynamic Orthogonal Small-Scales and Isogeometric Analysis

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ABSTRACT

In this talk we construct a stabilized finite element methods with correct energy behavior. To this purpose we employ isogeometric analysis [1] and dynamic orthogonal small-scales.

The classical stabilized methods, i.e. the streamline upwind Petrov-Galerkin method (SUPG), the Galerkin/least-squares method (GLS), and the variational multiscale method (VMS), are among the most popular finite element methods of the last decades. These methods are well-established but do not show correct energy behavior. We rectify this discrepancy and present our approach for the convection-diffusion equations [2] and the incompressible Navier-Stokes equations [3].

The convection-diffusion case demands the large- and small-scales to be H_0^1-orthogonal to arrive at a method with desired energy behavior. Its counterpart in case of the incompressible Navier-Stokes equations is to apply an orthogonality induced by the Stokes equations. This needs to be augmented with divergence-free H_0^1-small-scales to ensure correct energy evolution. The resulting formulation is of GLS-type and enjoys several favorable properties. These include (i) divergence-free solutions, (ii) the conservation of linear and angular momentum and (iii) divergence-free small-scales. A key observation is that the demand for correct energy behavior creates a link between the VMS, SUPG and GLS methodologies.

Computations display that the energy dissipation due to the small-scales can be negative in standard stabilized methods. A turbulent flow calculation shows improved energy behavior of the constructed method compared to the standard VMS method. The implementations use the IGA concept to ensure pointwise divergence-free solutions which is an important ingredient to get correct-energy evolution.

REFERENCES

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