Fast Divergence-Conforming Reduced Basis Methods for steady Navier-Stokes Flow

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ABSTRACT

Numerical methods and tools for simulating flow around complex geomeries have evolved rapidly in recent years. However, their usage usually requires access to high-performance computing facilities, which is not always feasible. New methods are required for an ever-increasing demand for computationally efficient desktop tools usable in real-time control, optimization and management. One such solution is dimensionality reduction through Reduced Order Modelling (ROM) based on Proper Orthogonal Decomposition (POD).[1]

We detail in this work an experiment involving two-dimensional stationary flow around a NACA0015 airfoil at various velocities and angles of attack. Two different methods are proposed: a run-of-the-mill reduced basis method based on a conventional Taylor-Hood high-fidelity solver, and one based on a divergence-conforming isogeometric flow solver. The reduced bases were constructed with POD and enhanced with supremizers[2] to stabilize the pressure field. We show how the latter method produces a fully divergence-free reduced basis, and how this property influences the linear systems by creating three-by-three block triangular matrices, a structure that can be exploited for significant speedups over the traditional matrix structure arising from the regular method.

REFERENCES

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[2] F. Ballarin, A. Manzoni, A. Quarteroni, and G. Rozza, Supremizer stabilization of POD-Galerkin approximation of parametrized steady incompressible Navier Stokes equations, *Int. J. Numer. Meth. Engng.*, Vol. **102**, pp. 1136–1161 (20125).