Construction and application of a dual isogeometric basis

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ABSTRACT

Finite element formulations look for the 'best' approximation of the solution of PDEs in a finite dimensional linear function space spanned by basis functions.

Instead of just one basis in which to expand all unknowns, we generally use a sequence of spaces which form a so-called DeRham complex, where every space in this complex is a linear vector space.

With each linear vector space $\mathcal{V}$, we can associate the algebraic dual space of linear functionals, $\mathcal{V}'$, acting on $\mathcal{V}$.

If the $\Phi_i(x)$ form a basis for $\mathcal{V}$, then we can define a (canonical) dual basis $\widetilde{\Phi}_j(x)$ for $\mathcal{V}'$ which satisfies, [1]

$$\langle \widetilde{\Phi}_j(x), \Phi_i(x) \rangle := \int_\Omega \widetilde{\Phi}_j(x)\Phi_i(x)\, \mathrm{d}\Omega = \delta_{ij} \ .$$

The use of a dual representation has several important advantages:

1. Part of the stiffness matrix becomes independent of the size and shape of the mesh. Deformation of the grid will not affect this part of the matrix;
2. Even for high order methods, parts of the stiffness matrix will remain very sparse due to the Kronecker-delta property
3. It will reduce the condition number of the stiffness matrix.

In this presentation the algebraic dual space for splines will be presented and its use in the solution of PDEs will be shown.

REFERENCES

[1] V. Jain, Y. Zhang, A. Palha and M. Gerritsma, *Construction and application of algebraic dual polynomial representations for finite element methods*, arXiv:1712.09472, (2017).