Robust Parallel Solvers for discontinuous Galerkin Space-Time Isogeometric Analysis of Parabolic Evolution Problems IGAA 2018

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ABSTRACT

In this talk, we construct and investigate fast solvers for large-scale linear systems of algebraic equations arising from the application of isogeometric analysis (IgA) to parabolic diffusion problems. We consider decompositions of the space-time cylinder into time slabs, where each slab is again decomposed into several space-time patches. We use dG techniques to provide information transfer between the time slabs, whereas the patches within a time slab are coupled in a conforming way. The considered discretization is based on a time-upwind scheme and gives optimal convergence rates for sufficiently smooth solutions, see [1].

The aim of the talk is to investigate fast solvers, which are based on the time parallel multigrid method, developed in [2]. We present two strategies for an efficient implementation of the smoother, which is the most costly part. One is directly based on the Isogeometric Tearing and Interconnecting (IETI-DP) method, a non-overlapping domain decomposition method utilizing the multipatch structure of the time slabs. This method has an excellent parallel scalability. For symmetric and positive definite problems it is proven that the iteration numbers are quasi-optimal with respect to the mesh-size. Numerical experiments also show the same behavior for the non-symmetric space-time matrices. Furthermore, we construct a preconditioner for approximating the smoother in such a way that only a preconditioner for the spatial problem is required. The main idea is a decomposition of the space-time matrix into a series of spatial problems via an eigendecomposition. The proposed algorithms are well suited for parallelization in time as well as in space. We conclude the talk with numerical experiments, confirming the theoretical results. Moreover, we present scalability studies, having parallelization in space and time simultaneously, up to several hundreds of cores on the RADON1 located at RICAM, Linz, Austria.

REFERENCES

[1] C. Hofer, M. Neumüller, U. Langer, and I. Toulopoulos, "Time-Multipatch Discontinuous Galerkin Space-Time Isogeometric Analysis of Parabolic Evolution Problems", *DK Computational Mathematics Linz Report Series*, Report No. 2017-**05**, (2017).

[2] M.J. Gander and, M. Neumüller, "Analysis of a new space-time parallel multigrid algorithm for parabolic problems", *SIAM Journal on Scientific Computing*, **38**, A2173–A2208 (2016).