Low-rank space-time decoupled isogeometric analysis for parabolic problems with varying coefficients

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ABSTRACT

In this work, we present a space-time isogeometric analysis scheme for the discretization of parabolic evolution equations with diffusion coefficients depending on both time and space variables. The discretization of this problem is based on the isogeometric method for parabolic equations with constant diffusion presented in [3]. In this approach, space and time are not discretized separately but time is considered as another variable. The time derivative appearing in the formulation of the model plays the role of an advection term in the last direction. The problem is considered in a space-time cylinder in $\mathbb{R}^{d+1}$ with $d = 2, 3$ and is discretized using higher-order and highly-smooth spline spaces.

We mainly focus on the computational challenges that appear in the matrix assembly step. The high polynomial degrees and the large supports of the basis functions make the matrix assembly task very challenging from a computational point of view. In order to overcome the complexity of the assembly, we apply the partial tensor decomposition method which was introduced in [2] for elliptic problems. By performing a low-rank decoupling of the operator into time and space components, the $d+1$-variate integrals that appear in the system matrix are replaced by a number of d-variate and univariate quadrature problems, thereby greatly reducing the computational effort of the assembly.

The method results in a complexity of $O(Rn^d p^{3d})$ for the quadrature step and a complexity of $O(Rn^{d+1} p^{d+1})$ for the generation of the global system matrix from the lower-variate integrals. Here, n and p are the dimensions and polynomial degrees of the discretization while R is the rank obtained in a low-rank approximation of the diffusion coefficient.

In our numerical experiments we demonstrate the efficiency of this approach for three-dimensional as well as for four-dimensional space-time domains. We present computation times for the assembly confirming the theoretical results as well as the error convergence of the discretization.

REFERENCES

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