Efficient Multigrid based solvers for Isogeometric Analysis

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ABSTRACT

Introduced in [1], Isogeometric Analysis (IgA) has become widely accepted in academia and industry. However, solving the resulting linear systems remains a challenging task. For instance, the condition number of the Poisson operator scales quadratically with the mesh width h, but, in contrast to standard Finite Elements, exponentially with the order of the approximation p [2]. The performance of (standard) iterative solvers thus decreases fast for higher values of p.

In this talk we propose an efficient solution strategy for IgA discretizations that is based on p-multigrid techniques used both as a solver and as a preconditioner in a Krylov subspace iteration method. The approach makes use of a hierarchy of B-spline based discretizations of different approximation orders, which is in contrast to (geometric) h-multigrid methods, where a hierarchy of coarser and finer meshes is constructed. The 'coarse grid' correction is determined at level $p = 1$, which enables us to use established solution techniques developed for low-order Lagrange finite elements. Prolongation and restriction operators are defined as mappings between arbitrary spline spaces, solely determined by the generating knot vectors, allowing us to combine coarsening in both h and p, leading to a flexible hp-multigrid.

Prelimenary numerical results are presented for different two-dimensional benchmark problems on non-trivial geometries. It follows from a Local Fourier Analysis [3], that the coarse grid correction and the smoothing procedure complement each other quite well. Moreover, the obtained convergence rates indicate that p-multigrid methods have the potential to efficiently solve IgA discretizations.

REFERENCES

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