Skeleton-stabilized immersed isogeometric analysis for incompressible flow problems

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ABSTRACT

Isogeometric Analysis (IGA) of incompressible flow problems has been an active topic of research over the last decade. In particular, IGA of mixed formulations for incompressible flow problems based on inf-sup stable velocity-pressure pairs has been demonstrated to be very suitable, which has led to the development of a range of isogeometric element families. In recent years, it has been shown that direct application of these element families in an immersed (Isogeometric Finite Cell) setting leads to local oscillations in the pressure field near cut boundaries.

In this contribution we present an alternative stabilization technique that avoids the stability problem observed for inf-sup stable velocity-pressure pairs in the Isogeometric Finite Cell method. The pivotal idea of the considered technique is to control the jump of high-order derivatives of the pressure field over the skeleton structure of the mesh. This skeleton-based stabilization technique allows utilizing identical discrete spaces for the velocity and pressure fields. To enable application of the skeleton-based stabilization technique in the Isogeometric Finite Cell setting, the system is complemented with a stabilization term for the velocity space similar to that of the pressure space. In contrast to the pressure stabilization, the velocity stabilization – which is referred to in the literature as Ghost-penalty stabilization – is only applied at the faces of the background mesh skeleton structure that are located near the cut boundaries.

Since the proposed skeleton-based stabilization technique is applicable in the conforming setting, we have studied its performance for a range of Stokes flow and moderate Reynolds number Navier-Stokes flow benchmark problems on two and three-dimensional conforming meshes, including the case of a multi-patch NURBS-based Isogeometric Analysis. We have observed the skeleton-based stabilization method to yield solutions that are free of pressure oscillations and velocity locking effects, and to yield optimal rates of convergence under mesh refinement. The observations for the conforming isogeometric setting extend to the immersed setting, where we have considered a range of two and three-dimensional problems for incompressible flows. To demonstrate the versatility of the proposed simulation strategy we have considered the Isogeometric Finite Cell analysis of Stokes flow through a porous medium, where the geometry is extracted directly from three-dimensional scan data.